

**AN INTRODUCTION TO LATENT GROWTH CURVE MODELING (LGC MODELING):  
A RESOURCE PACKET FOR PARTICIPANTS**

*Innovative Statistical Solutions for Evaluation Data:  
Latent Growth Curve Modeling to Examine Change Over Time*

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## Section 1: Introduction to Latent Growth Curve Modeling

### Definition of Latent Growth Curve Modeling

- ➔ Latent Growth Curve (LGC) modeling is an analytic technique within the structural equation modeling (SEM) framework to examine longitudinal change over time. LGC modeling allows evaluators to examine intra-individual change and inter-individual differences in intra-individual change, as well as examining both linear and non-linear change (i.e., quadratic or cubic growth).
- ➔ Because LGC Modeling is estimated within a structural equation Modeling (SEM) framework, a foundational knowledge of SEM is helpful for understanding the underlying features of LGC models. This review is outside of the scope of this resource packet. For further information, see these recommendation resources:

Byrne, B. M. (2012). *Structural Equation Modeling with Mplus: Basic concepts, applications and programming*. New York, NY: Taylor & Francis.

Kline, R. B. (2011). *Principles and Practice of Structural Equation Modeling*. New York, NY: Guildford Press.

### Assessing Differences Over Time: Types of Change

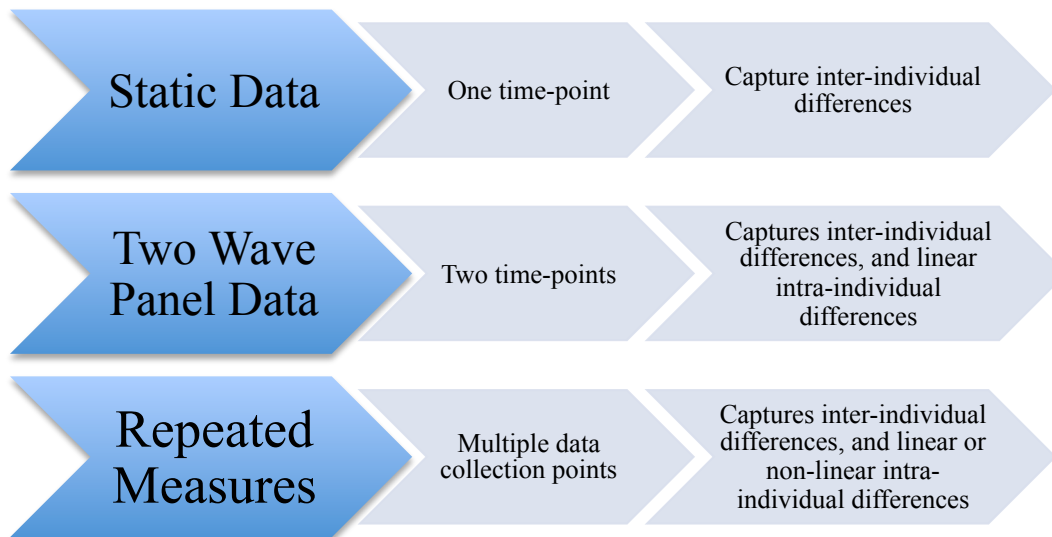
There are several differences between and within program participants that are relevant for evaluation design and practice.

- ➔ **Inter-individual differences** (*between* individuals):
  - Inter-individual differences are the differences *between* people that we observe at any given time point. For example, differences across participants in their level of knowledge at program enrollment.
- ➔ **Intra-individual change** (change *within* individuals over time):
  - Intra-individual change is the change *within* individuals that occurs over time (the same individual over time). For example, changes in knowledge or attitudes experienced by program participants during their participation in a program would be intra-individual change.
- ➔ **Inter-individual differences in intra-individual change** (*between* individual differences in change *within* individuals):
  - This idea is the combination of the previous two types of change. Inter-individual differences are differences between people in their rate of change over time. For example, over the course of program participation, participants may experience different difference levels of changes in knowledge over time.

## Traditional Approaches to Examining Change Over Time

There are several conventional statistical approaches to investigating change between and within evaluation participants, including static data, two-wave panel data, and repeated measures. These approaches have their unique advantages and disadvantages.

**Figure 1.** Features of Traditional Approaches to Examining Change Over Time



Although repeated measures statistical techniques, such as ANOVA, ANCOVA, or MANCOVA, can handle multiple data points, there is a growing recognition that these conventional approaches are inadequate for several reasons:

- ➔ These statistical approaches only provide information about the average rate of change for the overall sample and are incapable of capturing individual differences in change.
- ➔ These techniques assume that change in participants is linear.
- ➔ These statistical methods only analyze change in observed group means, individual differences in trajectories are treated as error variance.
- ➔ Finally, many of these statistical methods rely on the assumption that error variances of repeated measures are equal and independent, which is an assumption that is commonly violated with repeated measures data.

## Advantages of LGC Modeling Techniques

LGC modeling is a more sophisticated and rigorous approach to dealing with repeated measures data than these traditional approaches. LGC modeling is based on the analysis of both means and covariances, allowing the identification of both group effects in observed means, as well as individual effects observed in the covariances. LGC modeling techniques estimate the average rate of change of a sample over time, as well as the variability of that change rate within the sample. This is important because although we often want to understand the average change of participants in the programs we evaluate, we also want to understand individual variation in

change, so we can begin to answer questions about why some people experience changes in outcomes and others do not. Finally, unlike the previously mentioned statistical techniques, LGC modeling has the capability to explicitly assess and model measurement error.

## LGC Modeling & Program Evaluation

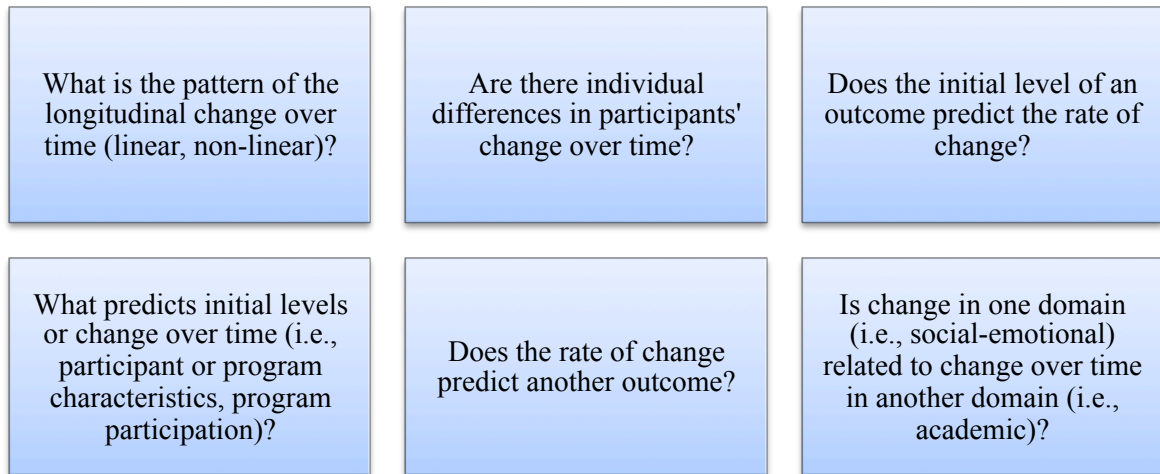
Latent growth curve modeling is a quantitative analytic technique that provides evaluators with the ability to sensitively assess how participants change over time by allowing evaluators to assess whether participants are changing in the same way (e.g., increasing, decreasing or not changing over time), whether the change is linear, and whether factors such as participant characteristics or program participation predicts change over time on program outcomes (Byrne, 2012).

LGC modeling techniques can enhance our evaluation practice by improving our ability to:

1. Address more nuanced questions about change over time. This allows us, as evaluators, to be more responsive to the information needs of our clients.
2. Accurately assess change over time in participants (i.e., examining both linear and non-linear change).
3. Understand participant differences in change over time and use predictors to explain heterogeneity of participants' development in program outcomes.
4. Examine change in more than one construct simultaneously (i.e., examine how participants' development in one outcome domain influences participants' development in other outcome domains)
5. Enhance the validity of evaluation findings through enhanced statistical rigor and control

Given that high standards for systematic inquiry are represented in the American Evaluation Association (AEA)'s guiding principles, evaluators should strive to increase the accuracy and clarity of evaluation findings. Our guiding principle of "systematic inquiry" urges us to "conduct systematic, data-based inquiries, adhering to the highest technical standards appropriate to the methods we use." As statistical tools and knowledge evolves, it is vital that we stay up to date and continuously adapt our analytic approaches to meet new standards of technical rigor. LGC modeling techniques are one example of sophisticated methods that have been developed to enable us to adhere to the field's highest technical standards. While there is increased recognition of the relevance of LGC modeling in psychology and other academic research circles, this is still a relatively new technique for the field of evaluation. As the field of quantitative methods advances, it is important that evaluators embrace new techniques available to improve evaluation practice and ensure the sustainability of the discipline.

Finally, these statistical techniques allow evaluators to answer more nuanced questions about change over time among program participants to better address the informational needs of the evaluand. As Figure 2 demonstrates, we can use LGC Modeling techniques to answer important evaluation questions, such as the questions outlined below.

**Figure 2. Example Evaluation Questions Addressed by LGC Modeling**

### Data Requirements/Assumptions: When can I conduct LGC Modeling?

There are several requirements that need to be met to conduct LGC modeling, including features of the data, sample size, and statistical assumptions.

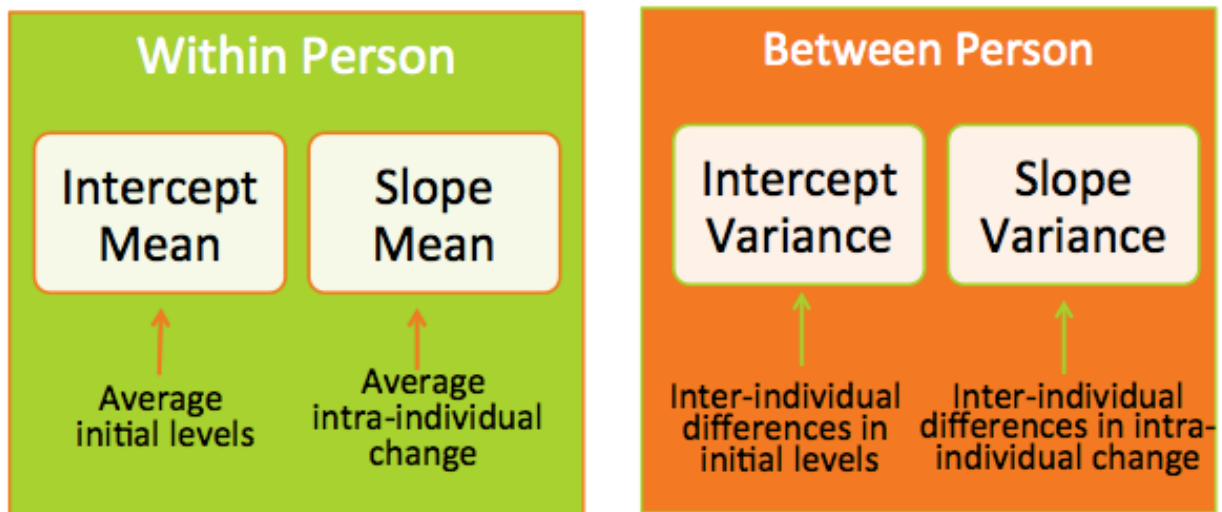
- ➔ **Data Requirements:** Collect at least 3 waves of data (repeated measures on 3 or more occasions) collected on a continuous dependent variable. Although the assessments do not need to be conducted at equally spaced intervals, all participants should be on the same assessment schedule. You can conduct a LGC model with only three data points, however the precision of your model increases with more waves of data. Additionally, measures should:
  - Be the same across waves of data collection
  - Not be standardized
  - Have strong psychometric properties (i.e., reliability, validity, factorial validity, measurement invariance<sup>1</sup>)
- ➔ **Sample Size Limitations:** Your sample size must be large enough for the detection of person-level effects. A good rule of thumb is  $N = 200$  or more per time point.
- ➔ **Statistical Assumptions:** Efforts should be made to meet the statistical assumptions for Structural Equation Modeling, however LGC Modeling is fairly robust to violations of all of these assumptions and you can explicitly test them in the model. The statistical assumptions include:
  - Multivariate normality
  - Homoscedasticity (error variances are equal across time)
  - Independence of errors (errors are uncorrelated)

<sup>1</sup> For a description of how to assess longitudinal invariance see <http://comm.eval.org/communities/resources/viewdocument/?DocumentKey=fda57328-2b30-4d48-9b41-6bd584072cf1>

## Latent Growth Curve Modeling Basics

The LGC model essentially has two core components: the within-person estimates and the between person estimates (see Figure 3). For each of these components, an intercept and a slope factor are estimated.

**Figure 3.** Core Components of Latent Growth Curve Models



- ➔ **WITHIN PERSON:** The within-person component of the model assesses intra-individual change in an outcome. This is the part of the model that assesses observed differences in group means.
  - Intercept Mean: The mean of the intercept is the average score of your outcome of interest at time one, adjusted for measurement error. This estimate answers questions such as, “What is the average level of life satisfaction in my sample at pre-test?”
  - Slope Mean: The mean of the slope is the average rate of change in your outcome of interest over time, adjusted for measurement error. This estimate answers questions such as, “What is my sample’s average annual rate of change in life satisfaction across three years?”
- ➔ **BETWEEN PEOPLE:** In contrast, the between people part of the model assesses inter-individual differences in change and is based on covariances.
  - Intercept Variance: The estimate for the variance of the intercept reflects the range of individual differences around the average initial level, and answers questions such as, “Are there individual differences in life-satisfaction at pre-test?”
  - Slope Variance: The variance of the slope provides information about inter-individual differences in change over time (intra-individual change). This estimate answers questions such as, “Are there individual differences in my samples’ development of life satisfaction over time?”

## Specifying a LGC Modeling for Estimation & LGC Modeling Figures

There are several elements of LGC modeling figures; some of these components are similar to the models developed for Structural Equation Modeling (SEM) and other components are unique (see Figure 4 for a visual representation).

**Intercept-Slope Covariance:** The intercept-slope covariance describes the relationship between the starting place or time 1 value (intercepts) and rate of change (slope). Because models slopes can be positive or negative, the intercept-slope covariance can be a little tricky because the interpretation of the covariance is dependent on the direction of our slope, as values can either increase or decrease over time.

➔ **For a Positive Covariance:** A positive covariance means that higher initial levels lead to higher positive rates of change, while lower initial levels relate to more negative rates of change.

- A positive intercept-covariance coupled with a positive slope means that higher initial levels are related to a steeper slope (or higher rate of change), and lower initial levels are related to slower increases or less steep positive slope.
- A positive intercept-covariance coupled with a negative slope means that higher initial levels are related to slower decline or less decline over time, and lower initial levels are related to steeper declines or steeper negative slope.

➔ **For a Negative Covariance:** A negative covariance means higher initial levels lead to more negative rates of change, while lower initial levels relate to more positive rates of change.

- A negative intercept-covariance coupled with a positive slope means that higher initial levels are related to a slower increase (or slower rate of positive change), and lower initial levels are related to steeper increases or steeper positive slope.
- A negative intercept-covariance coupled with a negative slope means that higher initial levels are related to steeper decline or more decline over time, and lower initial levels are related to slower declines or less steep negative slope.

**Intercept Factor Loadings:** The values you assign for the intercept must be constrained to be equal by setting them to the same value. Conventionally, these paths are constrained or “fixed” to “1.” The value you assign is arbitrary, however all paths must be constrained to the same number. All paths need to be equal at all time points, so the value of the intercept remains constant across time for each individual. This is because the intercept represents initial status and therefore cannot fluctuate over time.

**Slope Factor Loadings (Modeling Time):** The values you assign for the factor loadings from the slope to the manifest indicators tell the program how to interpret time. These loadings are our way of modeling time. The first slope path does not always have to represent the initial status. If your theory suggests that the true initial status is at the second assessment than you can put the zero there, thereby specifying this assessment as the true intercept. The other values have to represent a linear transformation of time (for linear models) so if you set your intercept to a second or third time points than the time points preceding the intercept have to be less than one, meaning they must have negative values.



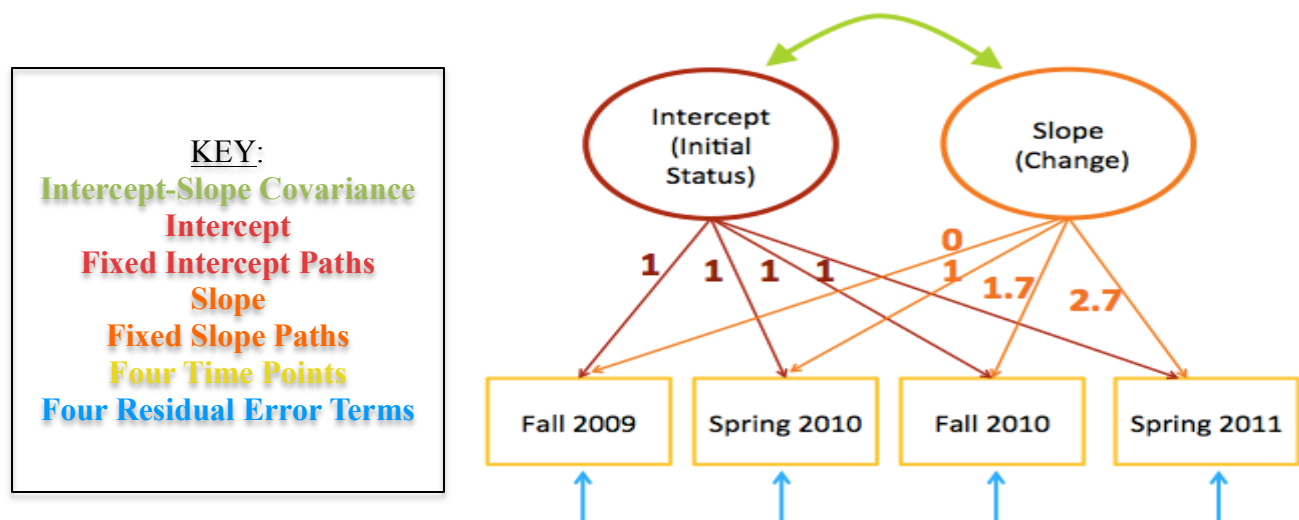
The constraint for “initial status” (intercept) will always be zero. Although the numbers assigned to the fixed loadings for the slope are arbitrary, the distance between these numbers represent the metric of time. For example, if you had three assessments that were each administered one-year apart, your value for the fixed loadings would be 0 (intercept), 1 (time two loading), and 2 (time three loading). With these loadings, an increase of “1” represents a year of time. In contrast, if your second assessment was administered six months after the first assessment and the third assessment was administered two years after the second assessment then your loadings would be 0 (intercept), 1 (time two loading), and 6 (time three loading). This is because in this example, the value of a “1” represents 6 months of time. Because the third assessment loading must be on the same metric of time as the second assessment loading, we divide the number of months between the first assessment and the third assessment, and then divide that value by the amount of time that passed between the first assessment and second assessment ( $36/6=6$ ).

**Quadratic Factor Loadings** (*Note: not featured in linear models*): Quadratic factor loadings are the squared value of your linear slope factor loadings. Given that LGC models should be parsimonious, linear slopes should always be assessed prior to quadratic slopes. If (1) a model with a linear slope is not an adequate representation of the data (i.e., poor model fit), (2) a model with a quadratic slope has significantly improved model fit, and (3) you have a significant mean for your quadratic factor then you have evidence to suggest that the change is quadratic and not linear.

**Observed Indicators:** The observed variables are the observed scores for the construct of interest at each of your data collection time points.

**Residual Error Terms:** The error variance associated with each of the observed indicators.

**Figure 4.** Example Latent Growth Curve Model Figure



## LGC Modeling with Predictors

LGC modeling techniques allow you to examine the role of predictors on your initial level and growth over time on the dependent variable. Latent growth curve models that include predictors are called conditional LGC models. Interpretations of model parameters are changed slightly for conditional models. This is because the interpretation of the average values for the intercept and slope are dependent on the other variables in the model, so in a sense it is interpreted in the same way you would interpret an intercept in regression, meaning that they are the average scores when everything else in the model is equal to zero. Furthermore, the interpretations of the slope variance and intercept variance also change, as they now represent the variation remaining in the intercept and slope after the variability in their prediction by the other variables in the model has been explained.

There are two types of predictors that you can incorporate into a LGC Model: (1) time varying predictors and (2) time invariant predictors.

- ➔ Time-Invariant Predictors: These predictors are measured at one time-point and thus cannot vary in the model. Examples of time invariant predictors include, ethnicity, whether or not the person participated in a program or specific component of a program, or the age of the person at entry into the program. To assess the association between these predictors and development in your construct you regress the slope and intercept on to the time-invariant predictors.
- ➔ Time Varying Predictors: This type of predictor varies over time and is measured at multiple time points. Examples of this include, constructs such as school engagement, life satisfaction, or motivation. Given that these predictors are also dependent upon time, they are a little more challenging to assess. One way to assess them is to conduct multiple simultaneous latent growth curve models, sometime referred to as parallel process models. For example, we could conduct a LGC model of self-efficacy and school engagement together and then look at the (1) covariance between the intercepts to see if self-efficacy at time one is related to school engagement at time one, (2) the covariance between the slopes to see if change in self efficacy is related to change in school engagement, (3) the covariance between the intercept of self-efficacy and the slope for school engagement to investigate if self-efficacy at time one is associated with change in school engagement and vice versa. There are other ways to model these time-varying predictors such as the use of definition variables or incorporating your time-varying predictors as repeated predictors of the outcome and you will find citations to articles that discuss this further in your resource packet.

## Section 2: Annotated Mplus Output

Below you will find annotated syntax and output for each level of measurement invariance. **Colored font** represents the standard commands used in Mplus syntax. Regular font represents the variable commands specific to this dataset. **Bolded text** represents descriptions of syntax and output.

## Syntax for Unconditional LGC Model

**TITLE:** LGC Model Self Efficacy –Title you gave your analysis.

**DATA: file is** longdata.dat; – Save your input file in the same folder as your data and enter the name of the dataset here.

**VARIABLE:** – This is the start of the command where you describe your variables.

**NAMES ARE** ID GEND SAFETY FALL09 SPRING10 FALL10 SPRING11; – All variables must be listed in the **SAME** order as listed in your database.

**USEVARIABLES ARE** FALL09 SPRING10 FALL10 SPRING11; – List only variables used in current analysis.

**MISSING=ALL** (999); – This is how you tell *Mplus* what your missing data code is.

**ESTIMATOR IS = MLR;** This is where you specify which estimator you want. MLR stands for full maximum likelihood with robust standard errors. This estimator is robust to departures from univariate and multivariate normality.

**MODEL:** – Enter model commands below here.

ISE **BY** Fall09@1 Name your intercept here (the name is arbitrary).  
 Spring10@1 The **BY** command tells *Mplus* what indicator  
 Fall10@1 variables your latent variable is measured by.  
 Spring11@1; The **@** command constrains or fixes the intercept loadings.

SSE **BY** Fall09@0 Name your slope variable here (the name is arbitrary).  
 Spring10@1  
 Fall10@1.7  
 Spring11@2.7;

ISE **WITH** SSE; – The “WITH” command specifies a covariance or correlation.

[ISE SSE]; – Here you are specifying that the intercept mean and slope mean should be estimated.

[Fall09-Spring11@0]; – You are constraining the intercepts of observed scores to be zero.

## “Short-Cut” Syntax for Unconditional LGC Model

**MODEL:**

ISE SSE | Fall09@0 Spring10@1 Fall10@1.7 Spring11@2.7; The | informs the program that you are estimating a LGC model.

### Optional Syntax

**OUTPUT: PATTERNS MODINDICES SAMPSTAT;** The “patterns” command requests all of the different missing variable patterns, the “modindices” command requests information on model parameters that are poorly estimated, the “sampstat” command requests sample statistics for all the variables in your model.

### Selected Annotated Output for Unconditional LGC Model

**COVARIANCE COVERAGE OF DATA** Covariance coverage is the proportion of the sample with data present across two variables. The default minimum coverage is .10 or 10%. Given that attrition is a common occurrence in longitudinal studies, this part of the output is important to examine when conducting latent growth curve models.

Minimum covariance coverage value 0.100

#### PROPORTION OF DATA PRESENT

	Covariance Coverage			
	Fall09	Spring10	Fall11	Spring11
Fall09	0.419			
Spring10	0.317	0.461		
Fall10	0.189	0.221	0.709	
Spring11	0.116	0.134	0.505	0.587

**Chi-Square Test of Model Fit-** The MLR estimator changes the distribution of the chi-square test. Because of this, when conducting chi-square difference tests you must use a correction. You can find more information about this on the *Mplus* website.

Value 8.919\*

Degrees of Freedom 5

P-Value 0.1123

Scaling Correction Factor for MLR 1.1688

**MODEL RESULTS-** Here we have the unstandardized estimates of our latent growth curve model. The loadings of the slope and intercept all have a p-value of .999 because they are constrained

	Estimate	Two-Tailed		
		S.E.	Est./S.E.	P-Value
ISE				
Fall09	1.000	0.000	999.000	999.000
Spring10	1.000	0.000	999.000	999.000
Fall10	1.000	0.000	999.000	999.000
Spring11	1.000	0.000	999.000	999.00
SSE				
Fall09	0.000	0.000	999.000	999.000
Spring10	1.000	0.000	999.000	999.000
Fall10	1.700	0.000	999.000	999.000
Spring11	2.700	0.000	999.000	999.00

SSE WITH ISE                      -0.050    0.023    -2.125    0.034 **Here we have the intercept-slope covariance.**

Means **Here we have the means for the intercept and slope.**

ISE	4.131	0.025	163.136	0.000
SSE	-0.076	0.013	-5.788	0.000

Intercepts **These are the intercepts that we constrained to 0**

Fall09	0.000	0.000	999.000	999.000
Spring10	0.000	0.000	999.000	999.000
Fall10	0.000	0.000	999.000	999.000
Spring11	0.000	0.000	999.000	999.000

Variances **Here we have the variances for the intercept and slope**

ISE	0.309	0.044	7.075	0.000
SSE	0.034	0.016	2.191	0.028

Residual Variances **Here is our residual error or measurement error**

Fall09	0.237	0.045	5.302	0.000
Spring10	0.331	0.026	12.755	0.000
Fall10	0.401	0.025	16.050	0.000
Spring11	0.325	0.039	8.281	0.000

### Selected Syntax for Conditional LGC Model

**MODEL:**

ISE SSE | Fall09@0 Spring10@1 Fall11@1.7 Spring11@2.7;

ISE SSE **ON** GENDER SAFETY; **Here we are regressing the intercept and slope onto our predictor variables (gender and safety)**

### Selected Output for Conditional LGC Model

#### MODEL RESULTS

ISE	ON				
SAFETY		0.163	0.051	3.191	0.001
GENDER		-0.168	0.052	-3.266	0.001

SSE	ON				
SAFETY		-0.025	0.026	-0.936	0.349
GENDER		0.046	0.027	1.716	0.086

SSE	WITH				
ISE		-0.049	0.024	-2.025	0.043

Intercepts **The means for the slope and intercept are now located here because the model is conditional and the slope and intercept are now endogenous or dependent variables.**

Fall09	0.000	0.000	999.000	999.000
Spring10	0.000	0.000	999.000	999.000
Fall10	0.000	0.000	999.000	999.000
Spring10	0.000	0.000	999.000	999.000
ISE	4.130	0.043	95.112	0.000
SSE	-0.088	0.022	-4.086	0.000

Residual Variances **The variances for the slope and intercept are now here.**

Fall09	0.230	0.045	5.106	0.000
Spring10	0.335	0.027	12.536	0.000
Fall10	0.393	0.025	15.752	0.000
Spring11	0.335	0.040	8.476	0.000
ISE	0.295	0.044	6.680	0.000
SSE	0.034	0.016	2.130	0.033

## Section 3: Software Options

### R

Website: <http://www.r-project.org/>

Online Support: <http://blog.revolutionanalytics.com/local-r-groups.html>

Pricing: Free

### Python

Website: <https://www.python.org/>

Online Support: <https://www.python.org/community/>  
Pricing: Free

### **EQS**

Website: <http://www.mvsoft.com/>  
Online support resources: <http://www.mvsoft.com/techsup.htm>  
Pricing: Free

### **Mplus**

Website: <http://www.statmodel.com/>  
Online Support: <http://www.statmodel.com/cgi-bin/discus/discus.cgi>  
Pricing: Lifetime membership with upgrades included (range represents different packages available). Student: \$195-\$350, University Pricing: \$595-895, Commercial/non-profit/government: \$695-1,095  
*Note:* Highly recommend by workshop facilitators, technical support from creators of software (usually within 24 hours).

### **Amos**

Website: <http://www-03.ibm.com/software/products/us/en/spss-amos/>  
Pricing: Approximately \$1,590 (exact pricing range unknown)  
*Note:* Rated highly for its graphical component

### **LISREL**

Website: <http://www.ssicentral.com/lisrel/>  
Online Support: <http://www.ssicentral.com/lisrel/resources.html>  
Pricing: Single User \$495, 12 month rental \$130

### **STATA**

Website: [www.stata.com](http://www.stata.com)  
Online Support: [www.stata.com/support](http://www.stata.com/support)  
Pricing: \$1,695 perpetual license

## **Section 4: Additional Resources**

Byrne, B. M. (2012). *Structural equation modeling with Mplus: Basic concepts, applications and programming*. New York, NY: Taylor & Francis.

Byrne, B. M., & Crombie, G. (2003). Modeling and testing change: An introduction to the latent growth curve model. *Understanding Statistics: Statistical Issues in Psychology, Education and the Social Sciences*, 2, 177-203.  
*Note:* Addresses missing data strategies for LGCM

Curran, P. J., Muthen, B. O., & Harford, T. C. (1998). The influence of changes in marital status on developmental trajectories of alcohol use in young adults. *Journal of Studies on Alcohol and Drugs*, 59, 647.

*Note:* Time-varying predictors example

Duncan, S. C., & Duncan, T. E. (1994). Modeling incomplete longitudinal substance use data using latent variable growth curve methodology. *Multivariate Behavioral Research*, 29, 313-338.

*Note:* Addresses missing data strategies for LGCM

Duncan, T. E., & Duncan, S. C. (2009). The ABC's of LGM: An introductory guide to latent variable growth curve modeling. *Social and Personality Psychology Compass*, 3, 979-991. doi:10.1111/j

Duncan, S. C., Duncan, T. E., & Stryker, L. A. (2006). An introduction to latent variable growth curve modeling: Concepts, issues, and applications (2<sup>nd</sup> ed.). Mahwah, NJ: Erlbaum.

Fan, X. (2003). Power of latent growth modeling for detecting group differences in linear growth trajectory parameters. *Structural Equation Modeling*, 10, 380-400.

Grimm, K.J. & Ram, N. (2009). Nonlinear growth models in Mplus and SAS. *Structural Equation Modeling*, 16, 676-701. doi:10.1080/10705510903206055

Grimm, K. J., & Ram, N. (2012). Growth curve modeling from a structural equation modeling perspective. In B. Laursen, T. D. Little, & N. A. Card (Eds.), *Handbook of Developmental Research Methods* (pp. 411-431). New York: Guilford Press.

Hu, L-T., & Bentler, P. M. (1995). Evaluating model fit. In R. H. Hoyle (Ed.), *Structural equation modeling: Concepts, issues, and applications* (pp. 76-99). Thousand Oaks, CA: Sage.

*Note:* Descriptions of model fit indices

Hu, L-T., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, 6, 1-55.

*Note:* Descriptions of model fit indices

Muthén, B., Kaplan, D., & Hollis, M. (1987). On structural equation modeling with data that are not missing completely at random. *Psychometrika*, 52, 431-462.

*Note:* Addresses missing data in SEM Frameworks

Preacher, K. J., Wichman, A. L., MacCallum, R. C., & Briggs, N. E. (2008). Latent growth curve modeling: Quantitative applications in the social sciences. Thousand Oaks, CA: Sage Publications.

Willett, J. B., & Keiley, M. K. (2000). Using covariance structure analysis to model change over time. In H. E. A. Tinsley & S.D. Brown (Eds.) *Handbook of applied multivariate statistics and mathematical modeling*, (pp. 665-694). San Diego, CA: Academic Press.

*Note:* Discussion of time-varying predictors



Willett, J. B., & Sayer, A. G. (1994). Using covariance structure analysis to detect correlates and predictors of individual change over time. *Psychological Bulletin*, *116*, 363-381.